

Lecture 8 Outline - The infinite well

- Solvay talk's: de Broglie and Schrödinger
- Quiz number 2...
- 1-D Schrödinger eqn stationary states [2.1]
- and let's get cracking on the infinite well [2.2]

Stationary States prescription

- Take a given *time-independent* potential $V(x)$

- Assume separation of variables:

$$\Psi(x, t) = \psi(x) \varphi(t) = \psi(x) e^{-iEt/\hbar}.$$

- Solve 1-D *time-independent* Schrödinger eqn:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi_n(x)}{dx^2} + V(x) \psi_n(x) = E_n \psi_n(x) .$$

- to get infinite number of solns: $\Psi_n(x, t) = \psi_n(x) e^{-iE_n t/\hbar}$.

- Assuming initial wave function at $\Psi(x, t = 0)$, we can solve for c_n such that $\Psi(x, 0) = \sum_{n=1}^{\infty} c_n \psi_n(x)$.

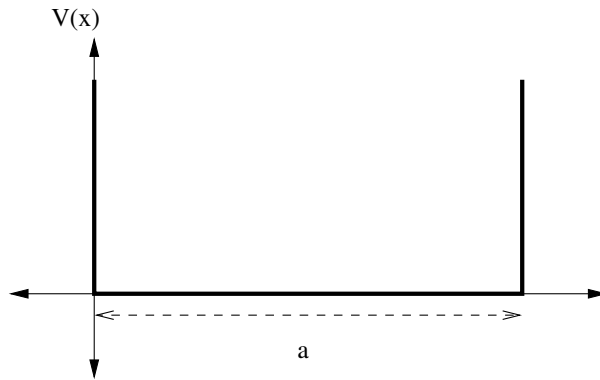
- The general soln to 1-D *time-dependent* Schrödinger eqn

$$\Psi(x, t) = \sum_{n=1}^{\infty} c_n \Psi_n(x, t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar}$$

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- First *time-independent* potential $V(x)$ to solve:

$$V(x) = 0, \quad \text{if } 0 \leq x \leq a \\ = \infty, \quad \text{elsewhere}$$



- this implies that $\psi_n(x) = 0$ for elsewhere
- while inside the well, we have $V(x) = 0$ and:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi_n(x)}{dx^2} = E_n \psi_n(x) .$$

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$$-\frac{\hbar^2}{2m} \frac{d^2\psi_n(x)}{dx^2} = E_n\psi_n(x) .$$

- Which is re-written as a simple harmonic oscillator eqn:

$$\frac{d^2\psi_n(x)}{dx^2} = -k_n^2\psi_n(x) \quad \text{where} \quad k_n = \frac{\sqrt{2mE_n}}{\hbar} .$$

- [assuming $E_n \geq 0$ (see Griffiths problem 2.2)].
- The SHO has the general solution:

$$\psi_n(x) = A_n \sin(k_n x) + B_n \cos(k_n x) .$$

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$$\psi_n(x) = A_n \sin(k_n x) + B_n \cos(k_n x) .$$

- To solve for coefficients impose boundary conditions,
- Assume that $\psi_n(x)$ is always continuous, and that $d\psi_n(x)/dx$ is continuous, except where $V(x) = \infty$:

$$\psi_n(0) = \psi_n(a) = 0$$

- that's at the edges, to which we match at $x = 0$:

$$\psi_n(0) = A_n \sin(k_n x) + B_n \cos(k_n x) = B_n = 0$$

- and matching at $x = a$:

$$\psi_n(a) = A_n \sin(k_n a) = 0$$

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$$\psi_n(a) = A_n \sin(k_n a) = 0$$

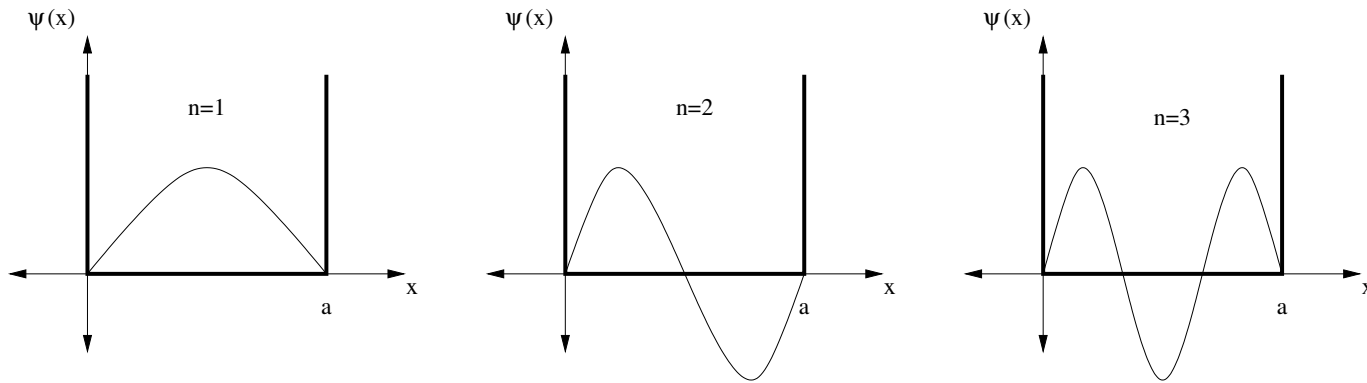
- So... either $A_n = 0$ (boring) or $\sin(k_n a) = 0$.

$$k_n a = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots$$

- but $k_n = 0$ is also boring since $\psi_n(x) = 0$.
- For the -ve solutions $\sin(-k_n a) = -\sin(k_n a)$ which we can simply absorb into A .
- Distinct solutions are: $k_n = n\pi/a$, with $n = 1, 2, 3$.

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$$\psi_n(a) = A_n \sin(k_n a) = 0 \quad \text{where} \quad k_n = n\pi/a$$



- Normalise: $\int_0^a |A_n|^2 \sin^2(k_n x) dx = |A_n|^2 \frac{a}{2} = 1$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right).$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$